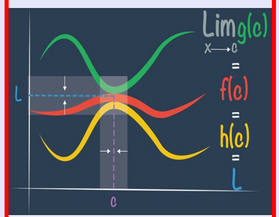
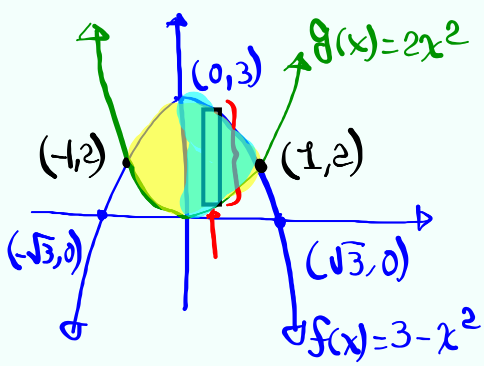
Calculus I

Lecture 49



Feb 19-8:47 AM

Find the area between $f(x) = 3 - x^2$ and $g(x) = 2x^2$.



$f(x) = g(x)$

$3 - x^2 = 2x^2 \qquad x^2 = 1$

$x = \pm 1$

Top $= 3 - x^2$

Bottom $= 2x^2$

Height = Top − Bottom

$= 3 - 3x^2$

width $= \Delta x$

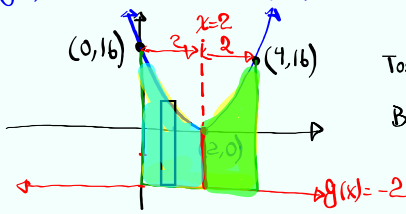
$$A = \int_{-1}^{1} [3 - 3x^2]\,dx$$

$$= 2\int_0^1 [3 - 3x^2]\,dx = 2\left[3x - x^3\right]\Big|_0^1 = 2\left[3(1) - 1^3 - 0\right] = 2(2) = \boxed{4}$$

Dec 2-7:27 AM

Find the area below $f(x)=(2x-4)^2$, above $g(x)=-2$ for $0\le x\le 4$.

$2x-4=0$
$x=2$



Top $=(2x-4)^2$

Bottom $=-2$

Height $=(2x-4)^2-(-2)$

width $=\Delta x$

$$A=\int_0^4\left[(2x-4)^2+2\right]dx=2\int_0^2\left[(2x-4)^2+2\right]dx$$

$$=2\int_0^2\left[4x^2-16x+18\right]dx=2\left[\frac{4x^3}{3}-\frac{16x^2}{2}+18x\right]\Bigg|_0^2$$

$$=2\left[\frac{4\cdot 2^3}{3}-8(2)^2+18(2)-0\right]=2\left[\frac{32}{3}-32+36\right]$$

$$=2\left[\frac{32}{3}+4\right]$$

$$=2\left[\frac{32}{3}+\frac{12}{3}\right]=2\cdot\frac{44}{3}$$

$$\boxed{\frac{88}{3}}$$

Dec 2-7:35 AM

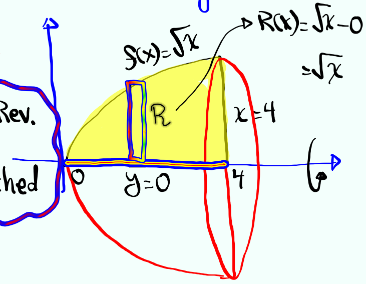
Volume of revolution.

Suppose we rotate the area bounded by $f(x)=\sqrt{x}$, $y=0$, and $x=4$ by x-axis.

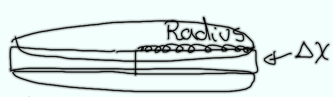
1) Draw the region.

Ref. Rec. $\perp$ Axis of Rev.

Region is totally attached to axis of Rev.



Disk Method



$$V=\int_a^b \pi\left[R(x)\right]^2dx$$

$$V=\int_0^4\pi\left[\sqrt{x}\right]^2dx=\pi\int_0^4 x\,dx$$

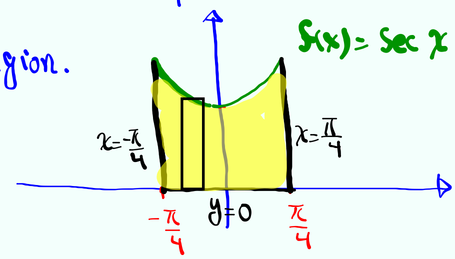
$$=\pi\cdot\frac{x^2}{2}\Bigg|_0^4$$

$$=\frac{\pi}{2}\left[4^2-0^2\right]=\frac{\pi}{2}\cdot 16=\boxed{8\pi}$$

Dec 2-7:46 AM

Consider the region bounded by $f(x) = \sec x$, $y = 0$, $x = -\frac{\pi}{4}$, and $x = \frac{\pi}{4}$.

1) Draw the region.



2) Find its area.

$$A = \int_{-\pi/4}^{\pi/4} (\sec x - 0)\,dx = 2\int_0^{\frac{\pi}{4}} \sec x\,dx = 2\left(\ln(\sec x + \tan x)\right)\Big|_0^{\frac{\pi}{4}}$$

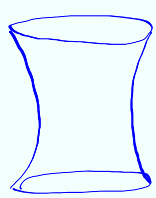
$$= 2\left[\ln\left(\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right) - \ln(\sec 0 + \tan 0)\right]$$

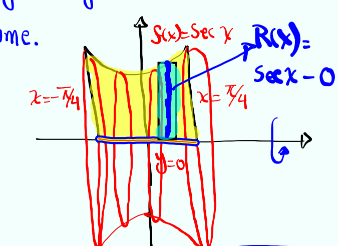
$$= 2\left[\ln(\sqrt{2} + 1) - \ln(1 + 0)\right]$$

$$= \boxed{2\ln(\sqrt{2} + 1)}$$

Dec 2-7:56 AM

3) Rotate the region by x-axis and find its volume.





1) Region is totally attached to axis of Rev.
2) Ref. Rec. $\perp$ axis of Rev.

Disk Method

$$V = \int_a^b \pi[R(x)]^2\,dx = \int_{-\pi/4}^{\pi/4} \pi[\sec x - 0]^2\,dx$$

$$= 2\pi\int_0^{\pi/4} \sec^2 x\,dx$$

$$= 2\pi \tan x\Big|_0^{\pi/4} = 2\pi\left[\tan\frac{\pi}{4} - \tan 0\right]$$

$$= 2\pi \cdot 1$$

$$= \boxed{2\pi}$$

Dec 2-8:06 AM

Suppose $f(x) = \int_{u(x)}^{v(x)} g(t)\,dt$

$u(x)$ & $v(x)$ must be diff.
$g(t)$ must be Cont.

$$f'(x) = g(v(x)) \cdot v'(x) - g(u(x)) \cdot u'(x)$$

$$f(x) = \int_1^{x^2} \sin(\sqrt{t})\,dt$$

$$f'(x) = \sin(\sqrt{x^2}) \cdot 2x - \sin(\sqrt{1}) \cdot 0$$

$$f'(x) = 2x \sin x \quad , \; x \geq 0$$

Dec 2-8:15 AM

$$f(x) = \int_0^x \frac{1}{t^2+1}\,dt$$

1) $f(0) = \int_0^0 \frac{1}{t^2+1}\,dt = 0$

2) $f'(x) = \frac{1}{x^2+1} \cdot 1 - \frac{1}{0^2+1} \cdot 0$

$f'(x) = \frac{1}{x^2+1} > 0 \rightarrow f(x)$ is increasing

3) $f''(x) = \frac{0 \cdot (x^2+1) - 1 \cdot 2x}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2}$

P.I.P.
$f''(x) = 0$ or und.
$x = 0$

4)

x	$-\infty$	0	∞
$f'(x)$	$+$		$+$
$f''(x)$	$+$	•	$-$
$f(x)$	↗		↗

I.P.
$(0, 0)$

Use technology and graph $f(x) = \int_0^x \frac{1}{t^2+1}\,dt$ and verify the sign chart.

Dec 2-8:22 AM